Text

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Expanding the least squares error by applying the affine transformation definition:

The gradient of the least squares error with respect to each parameters of the transformation is:

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* 
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Setting all the partial derivatives in 1a to zero, we get a system of 6 equations:

(1)   
(2)   
(3)   
(4)   
(5)   
(6) 

The matrix form of the equation **Sh = u** becomes:

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The identities in the S matrix can be calculated as follows:



Likewise, the identities in the u vector can be calculated as:



Finally, the optimal parameters of the affine transformation are:

*  =>  (answer)

Text, letter

Description automatically generatedLet denote the vectors as follows:  
Then, the formula to recover the rotation angle from the corresponding unit vectors is:  (answer)

Text, letter

Description automatically generatedThe scale factor as the ratio of the norms is:   
 (answer)

Text, letter

Description automatically generatedThe equation (1) is:   
  
Then we derive the translation =>  (answer)

Text, letter

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From the points, the vectors are constructed as:



Calculating the angle:



Calculating the scale:



Pick a random endpoint, which is 

Calculare the translation:



Finally, the transformation becomes:

 (answer)